Relative Motion

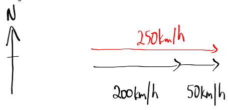
Relative Motion problems involve the addition of velocity vectors.

$${}_p\vec{V}_g = {}_p\vec{V}_a + {}_a\vec{V}_g$$

to observer on ground | airspeed + heading | wind

SP1

a) heading [E]

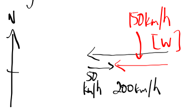


$${}_p\vec{V}_g = {}_p\vec{V}_a + {}_a\vec{V}_g$$

${}_p\vec{V}_g$ = 200km/h [E] + 50km/h [E]

${}_p\vec{V}_g$ = 250km/h [E]

b) heading [W]



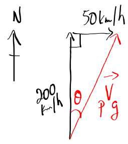
$${}_p\vec{V}_g = {}_p\vec{V}_a + {}_a\vec{V}_g$$

${}_p\vec{V}_g$ = 200km/h [W] + 50km/h [E]

${}_p\vec{V}_g$ = 200km/h [W] − 50km/h [W]

${}_p\vec{V}_g$ = 150km/h [W]

c) heading [N]



$${}_p\vec{V}_g = {}_p\vec{V}_a + {}_a\vec{V}_g$$

${}_p\vec{V}_g$ = 200km/h [N] + 50km/h [E]

${}_p\vec{V}_g$ = 206km/h [N14°E]

MUST SOLVE USING A VECTOR ADDITION DIAGRAM

$c^2 = a^2 + b^2$

$c^2 = 200^2 + 50^2$

c = 206 km/h ← only the magnitude

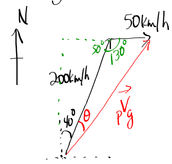
$\tan\theta = \frac{50}{200}$

θ = 14°

The velocity of the plane with respect to the ground is:

206km/h [N14°E]

d) heading [N40°E]



$${}_p\vec{V}_g = {}_p\vec{V}_a + {}_a\vec{V}_g$$

${}_p\vec{V}_g$ = 200km/h [N40°E] + 50 km/h [E]

${}_p\vec{V}_g$ = 235km/h [N49°E]

Solve with a vector addition diagram

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 200^2 + 50^2 - 2(200)(50)(\cos 130°)$

c = 235 km/h ← magnitude

Law of Sines

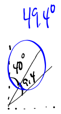
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{235}{\sin 130°} = \frac{50}{\sin\theta}$$

$$\sin\theta = \frac{50\sin 130°}{235}$$

θ = 9.4°

49.4°

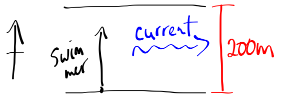


The velocity of the plane wrt the ground is

235km/h [N 49°E]

(40 + 9.4°)

2.



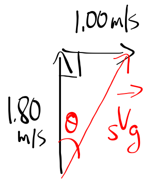
$$\vec{V}_{sg} = \vec{V}_{sw} + \vec{V}_{wg}$$

$\vec{V}_{sg}$: to observer on riverbank; $\vec{V}_{sw}$: swimspeed + heading; $\vec{V}_{wg}$: current

$$\vec{V}_{sg} = 1.80\,m/s\,[N] + 1.00\,m/s\,[E]$$

$$\vec{V}_{sg} = ??$$

must use a vector addition diagram.



$$c^2 = a^2 + b^2$$
$$c^2 = (1.80)^2 + (1.00)^2$$
$$c = 2.06\,m/s$$ ← magnitude

$$\tan\theta = \frac{1.00}{1.80}$$
$$\theta = 29.1^\circ$$

The velocity of the swimmer wrt to the riverbank is:

2.06 m/s [N 29.1°E]

b) how long to reach the opp. side of river?

$$\vec{V} = \frac{\Delta\vec{d}}{\Delta t}$$

* the directions for $\Delta\vec{d}$ and $\vec{V}$ MUST match

$$\vec{V}_{north} = \frac{\Delta\vec{d}_{north}}{\Delta t}$$

$$\Delta t = \frac{\Delta\vec{d}_{north}}{\vec{V}_{north}}$$

$$\Delta t = \frac{200\,m\,[N]}{1.80\,m/s\,[N]}$$

time to cross the river → $\Delta t = 111\,s$

c) how far downstream? [E]

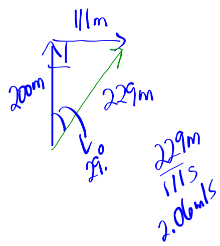
$$\vec{V}_{east} = \frac{\Delta\vec{d}_{east}}{\Delta t}$$

$$\Delta\vec{d}_{east} = \vec{V}_{east}\,\Delta t$$

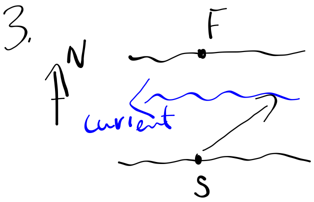
$$\Delta\vec{d}_{east} = (1.00\,m/s\,[E])(111\,s)$$

$$\Delta\vec{d}_{east} = 111\,m\,[E]$$

Land 111 m downstream



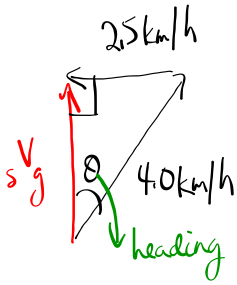
3.



$$\vec{V}_{sg} = \vec{V}_{sw} + \vec{V}_{wg}$$

to observer on ground. | swimspeed + heading | current

$$? \, [N] = 4.0 \text{ km/h} \, [??] + 2.5 \text{ km/h} \, [W]$$



a) In what direction must the swimmer head in order to go straight across?

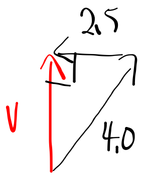
$$\sin\theta = \frac{2.5}{4.0}$$

$$\theta = 39^\circ$$

The swimmer must head [N 39° E] in order to go straight across

b) If the river is 2.0 km wide, how long to cross?

↑[N] ← must use the "north" velocity.



$$c^2 = a^2 + b^2$$

$$4.0^2 = 2.5^2 + b^2$$

$$b = 3.1 \text{ km/h}$$

$$\vec{V} = \frac{\Delta \vec{d}}{\Delta t}$$ (north, north)

$$\Delta t = \frac{\Delta \vec{d}}{\vec{V}}$$

$$\Delta t = \frac{2.0 \text{ km} \, [N]}{3.1 \text{ km/h} \, [N]}$$

$$\Delta t = 0.64 \text{ h}$$

(38 min)